SUNSHINE JIANG & WENQI DING

MATH WITHOUT NUMERS

18.000

WHO ARE WE?

Sunshine Jiang

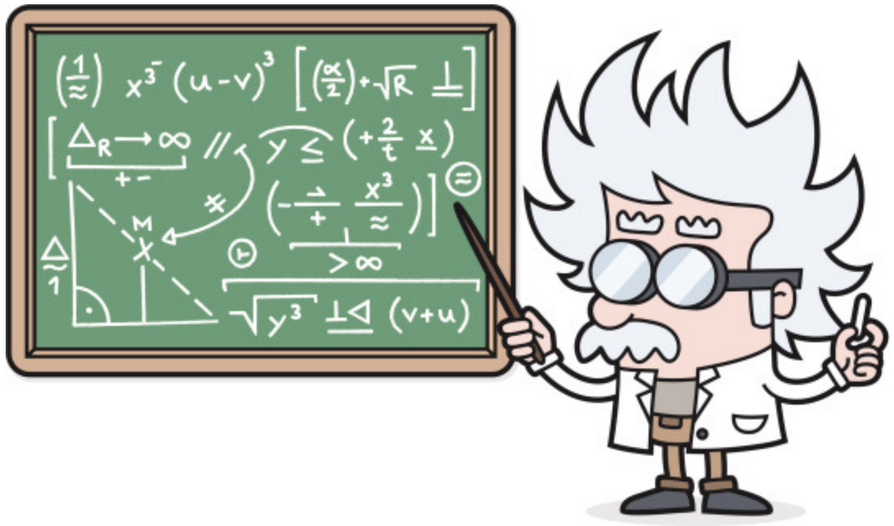
- **Freshman, potential physics & math and computer science major**

Wenqi (Helen) Ding

- **Freshman, potential computer science major**

ANSWER QUESTIONS AND WIN PRIZES!


YES



SUNSHINE JIANG 2021.11.20

FUN TOPOLOGY

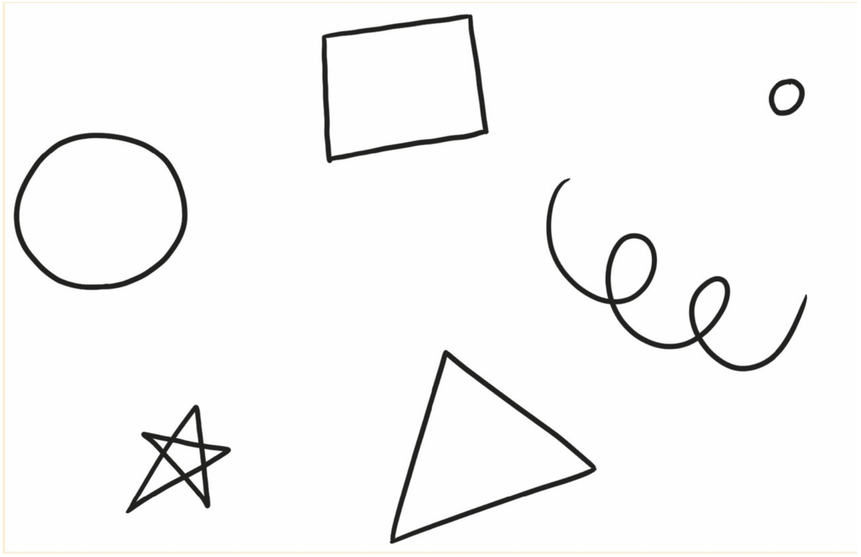
Math Without Numbers

SHAPES

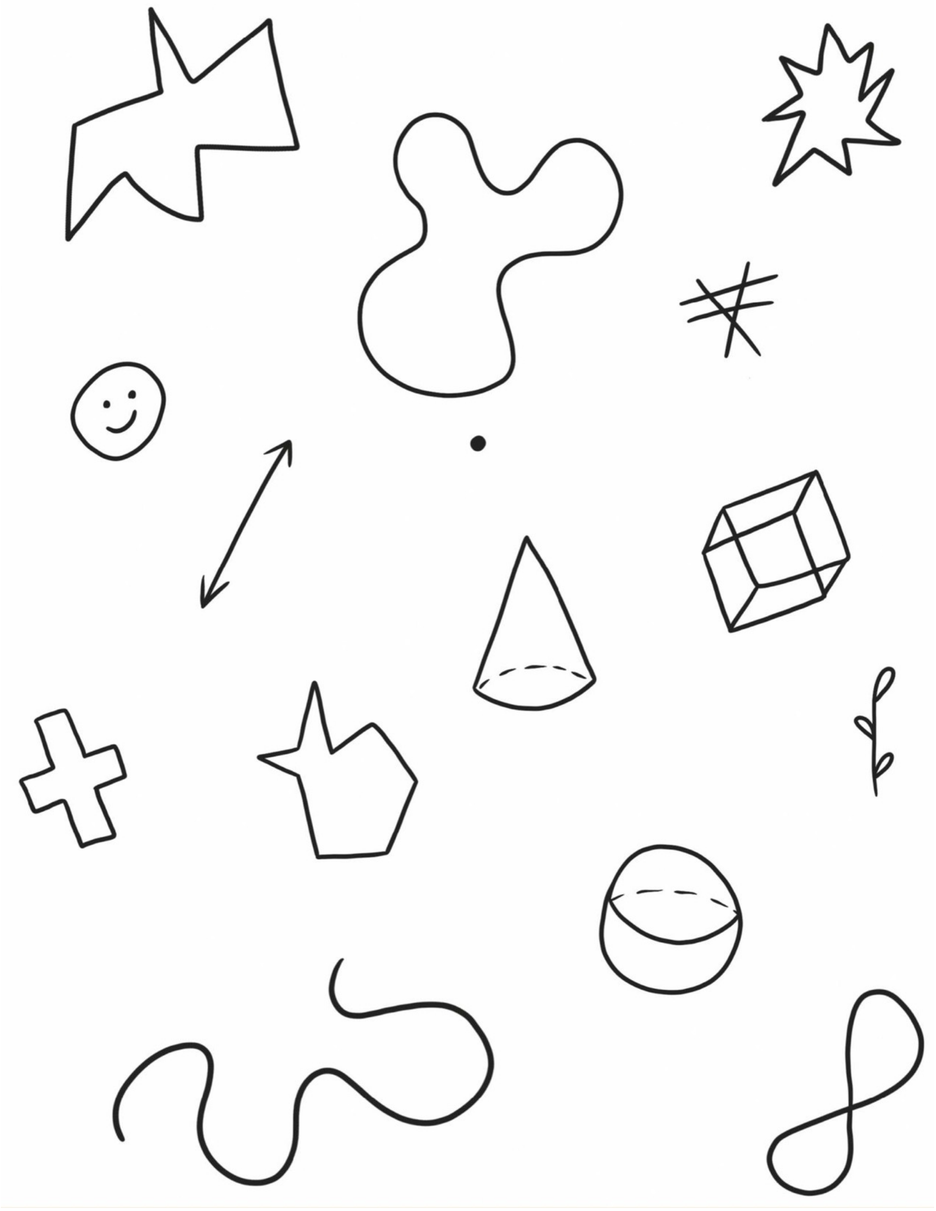
HOW MANY SHAPES THERE ARE?

GENERALIZED POINCARÉ CONJECTURE

DRAW SHAPES



DEFINITION!



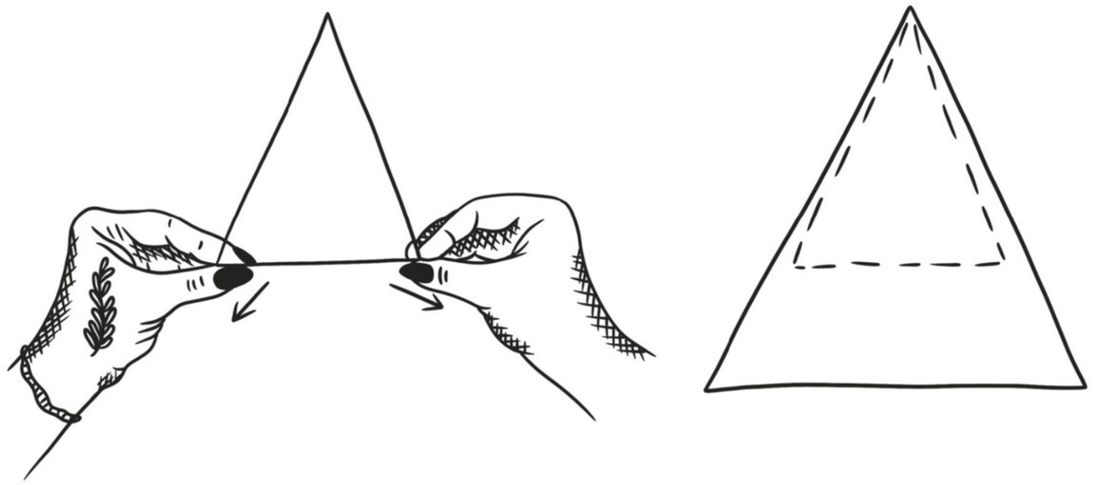
CONGRUENT

- **Geometry:** Two mathematical figures have the same geometry if the figures have the **same shape and congruent measures.**

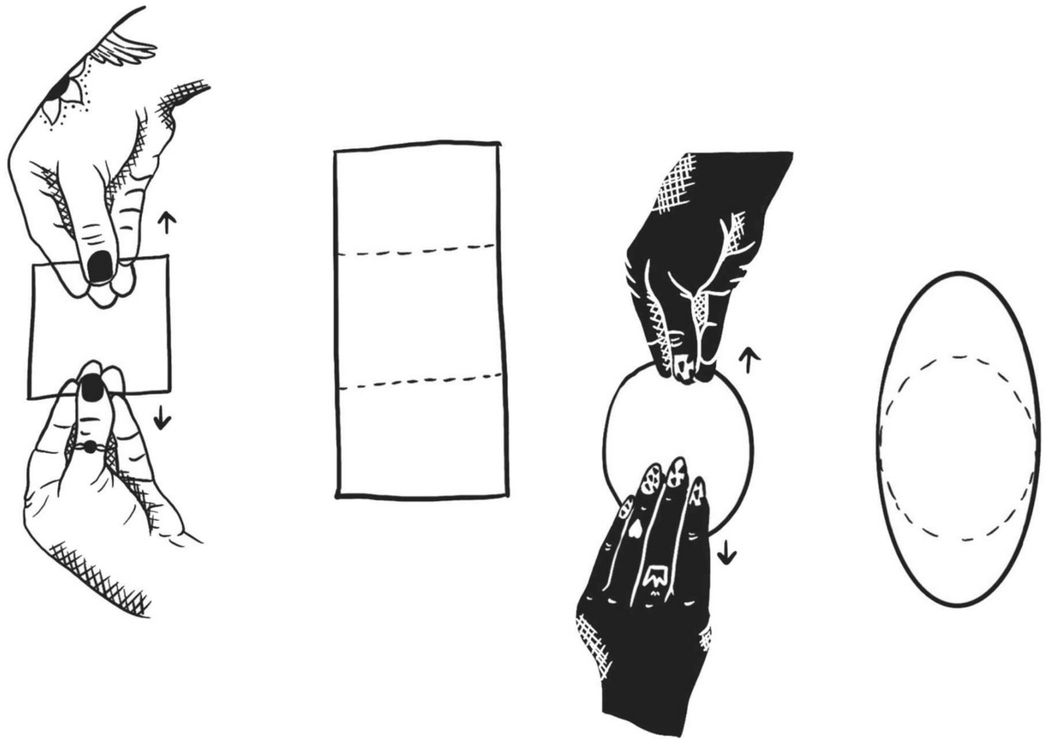
- **Topology:** Two mathematical figures have the same topology **if one figure can be transformed into the other figure by twisting and stretching, without ripping or gluing.**

- **Shapes made out of thin, endless stretchy material**

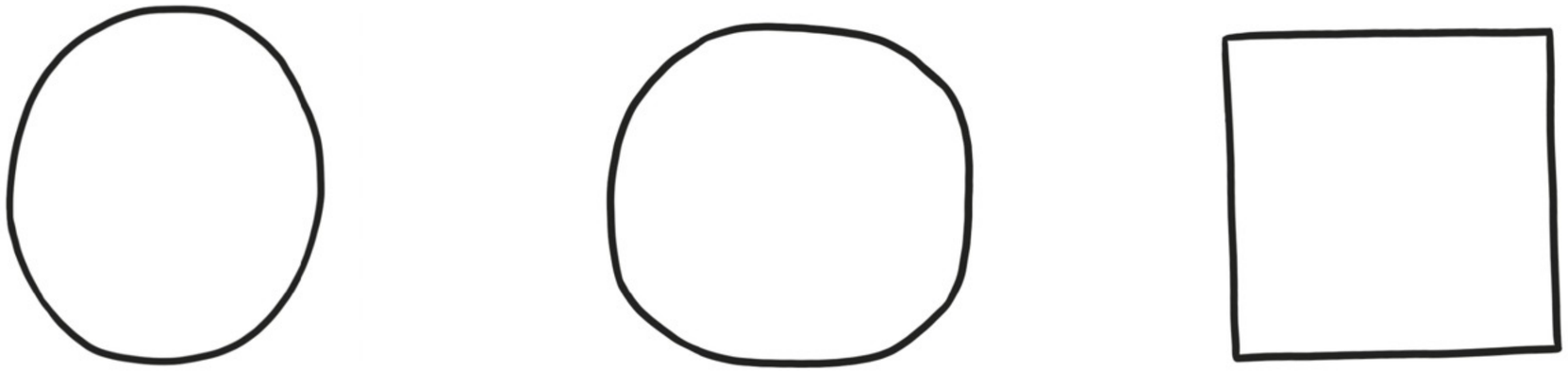
SIZE DOES NOT MATTER



CIRCLE=OVAL

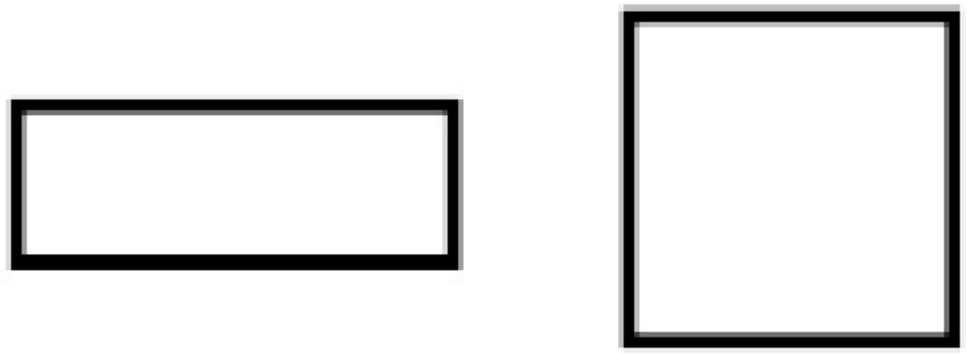


CIRCLE=SQUARE



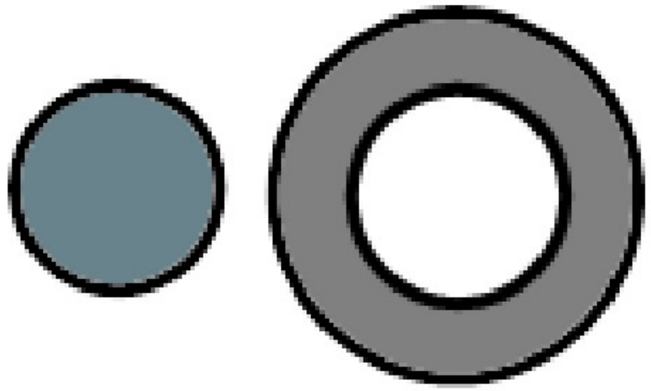
GEOMETRY/ TOPOLOGY/NEITHER

- **topology**



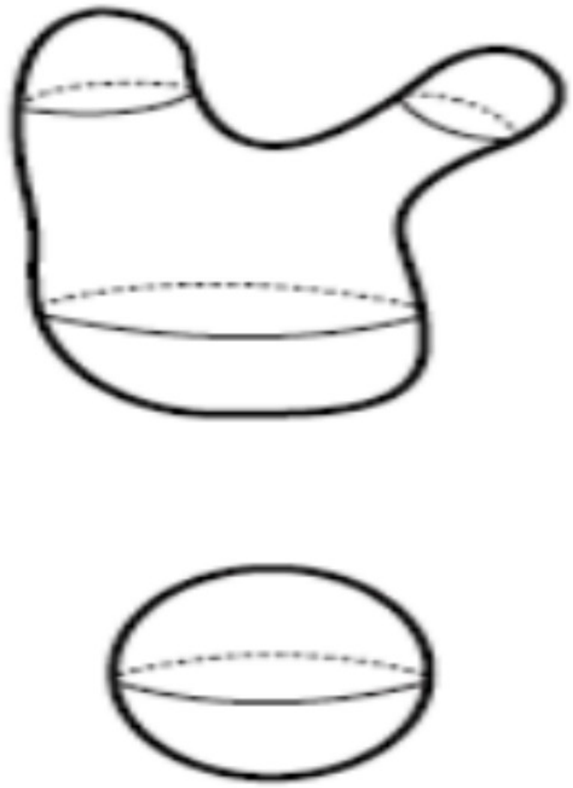
GEOMETRY/ TOPOLOGY/NEITHER

- **neither**



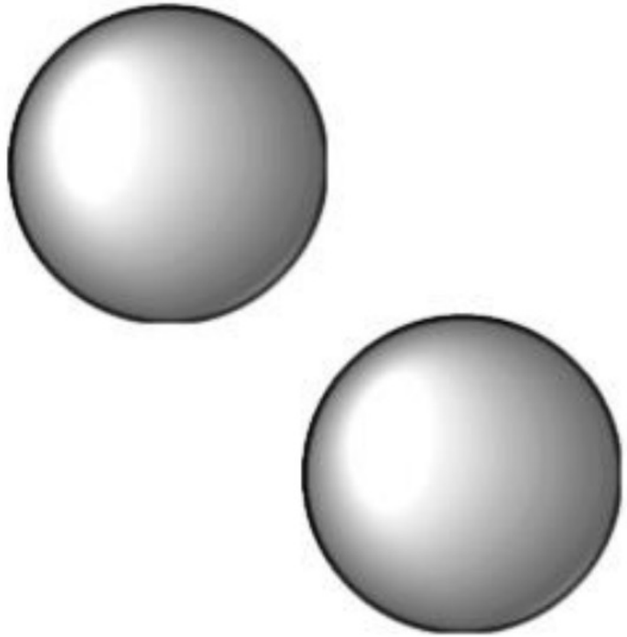
GEOMETRY/ TOPOLOGY/NEITHER

- **topology**



GEOMETRY/ TOPOLOGY/NEITHER

- **geometry**



GEOMETRY/ TOPOLOGY/NEITHER

- **topology**

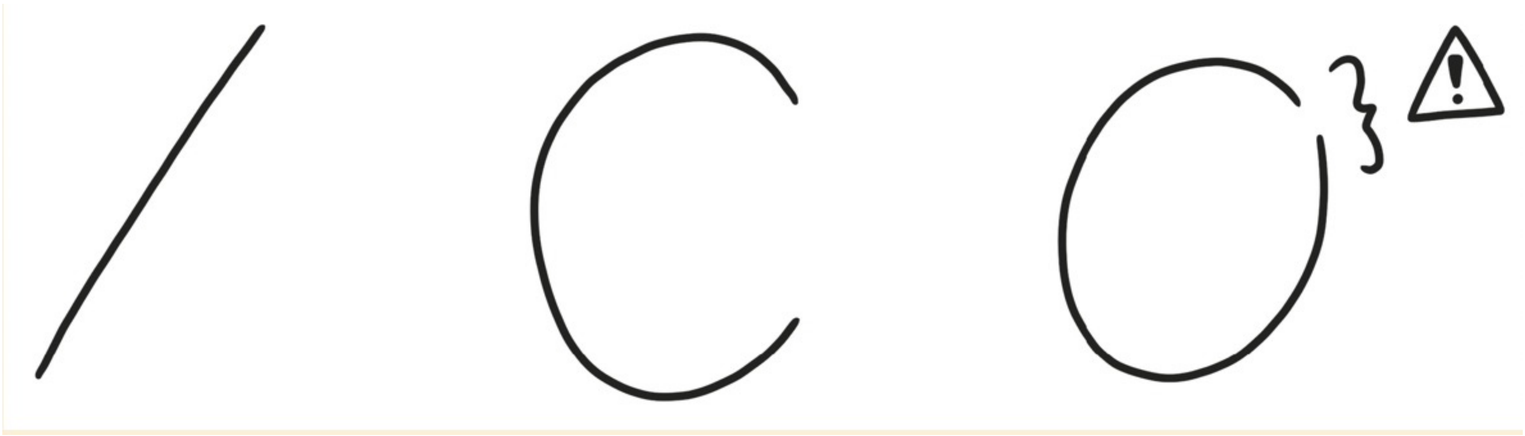
WHAT IS THE SHAPE OF A NECKLACE

- **We focus only on the core, underlying shape: the basic features that make a shape the shape it is.**

- **S-1**

OTHER SHAPES?

LINE



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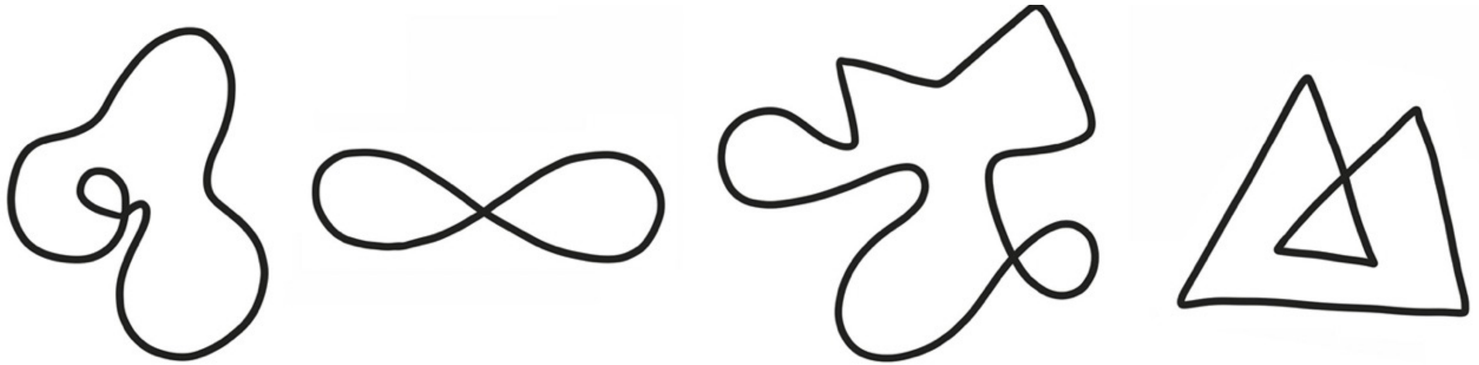
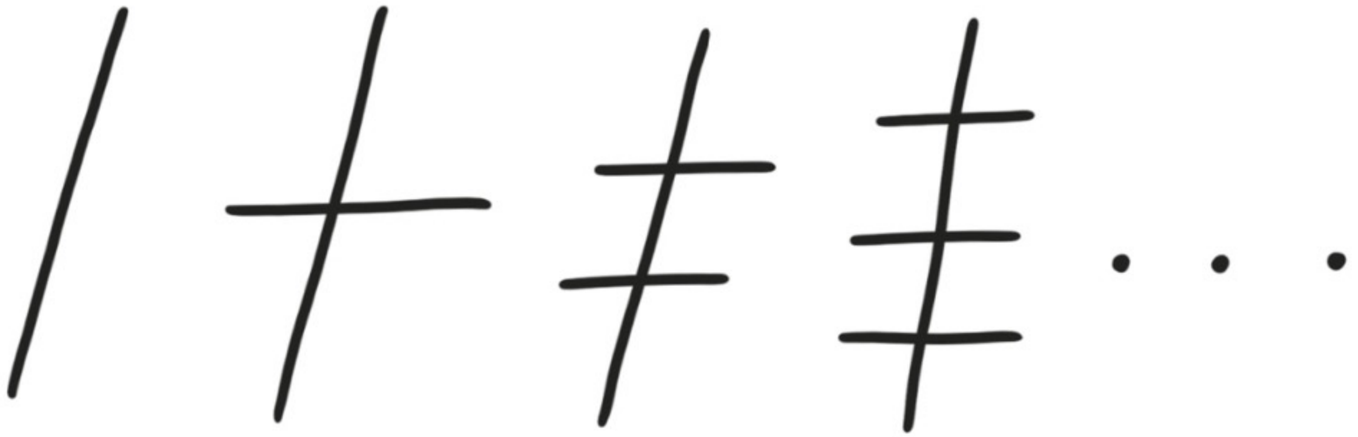
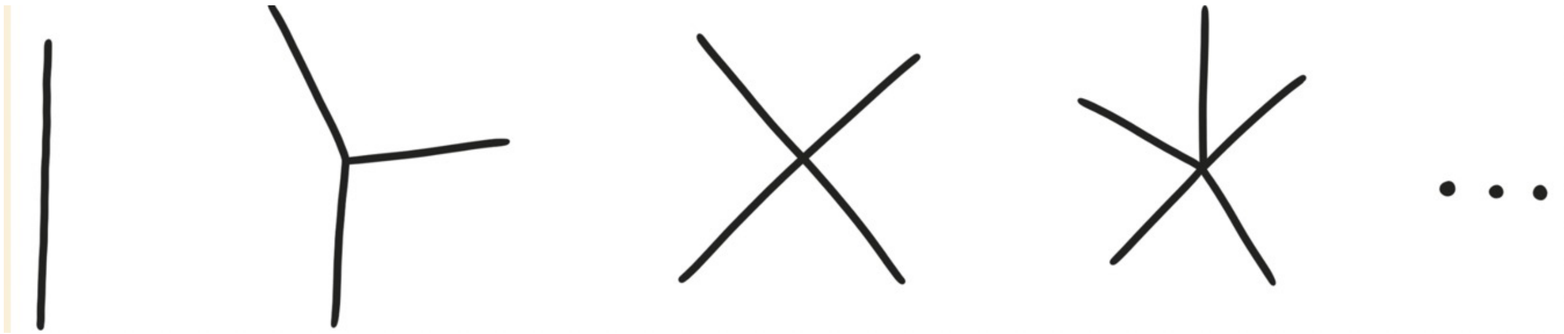
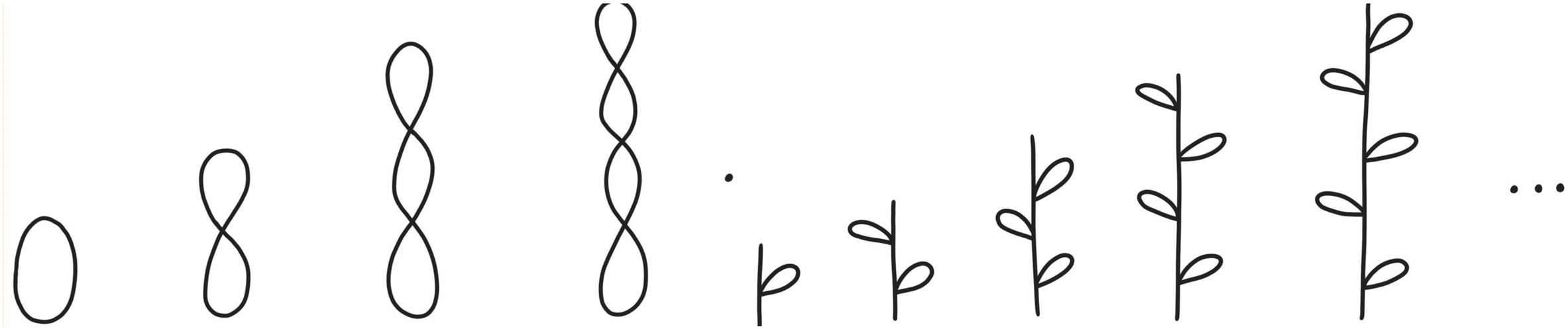


HOW MANY SHAPES THERE ARE?

INFINITY!







INFINITY FAMILY

- **Prove infinitely many of something**
- **describe a systematic process that keeps creating new different examples of that thing.**

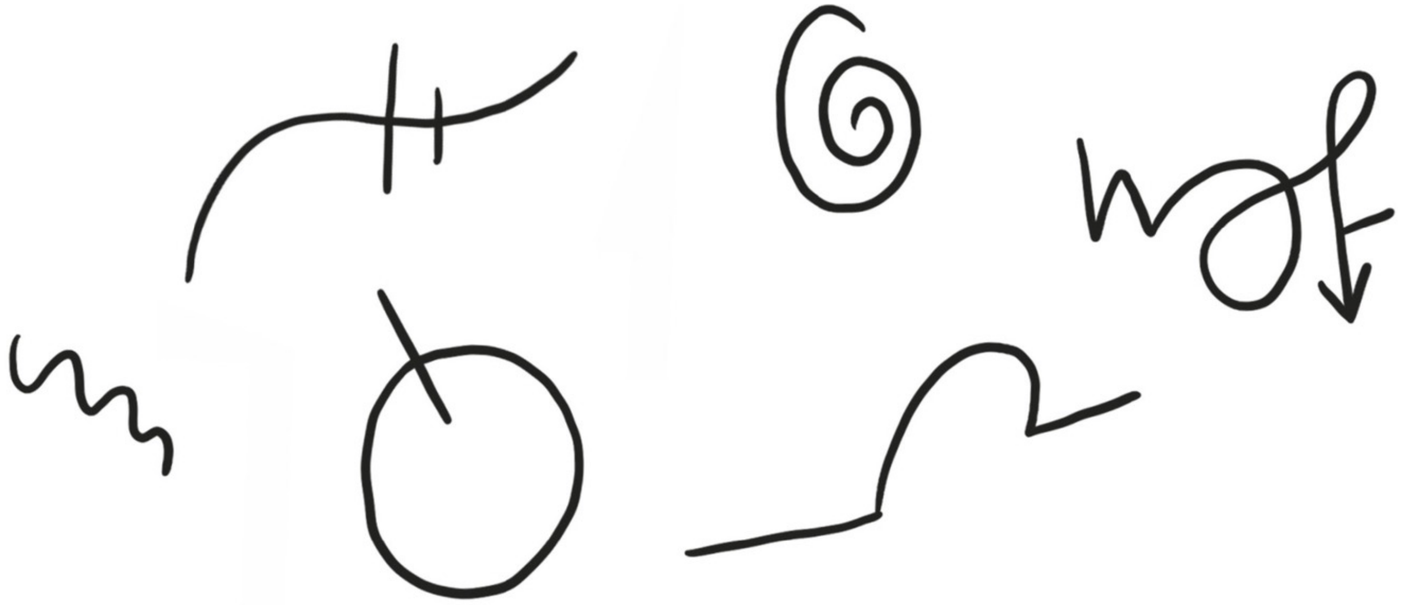
BETTER QUESTION

MANIFOLDS

DEFINITION

- **No special points, no end-points, no crossing-points, no edge-points, no branching-points.**
- **It has to be the same everywhere.**

- **Dimension: the material it is made out of**

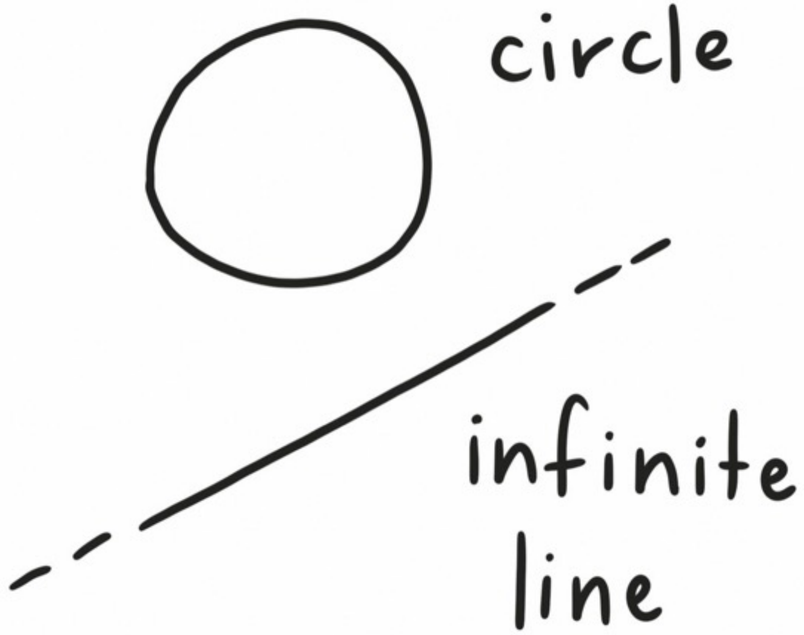


MAYBE EXACT CATEGORIES OF MANIFOLDS

ONE-DIMENSIONAL MANIFOLD?

MADE BY STRING-LIKE

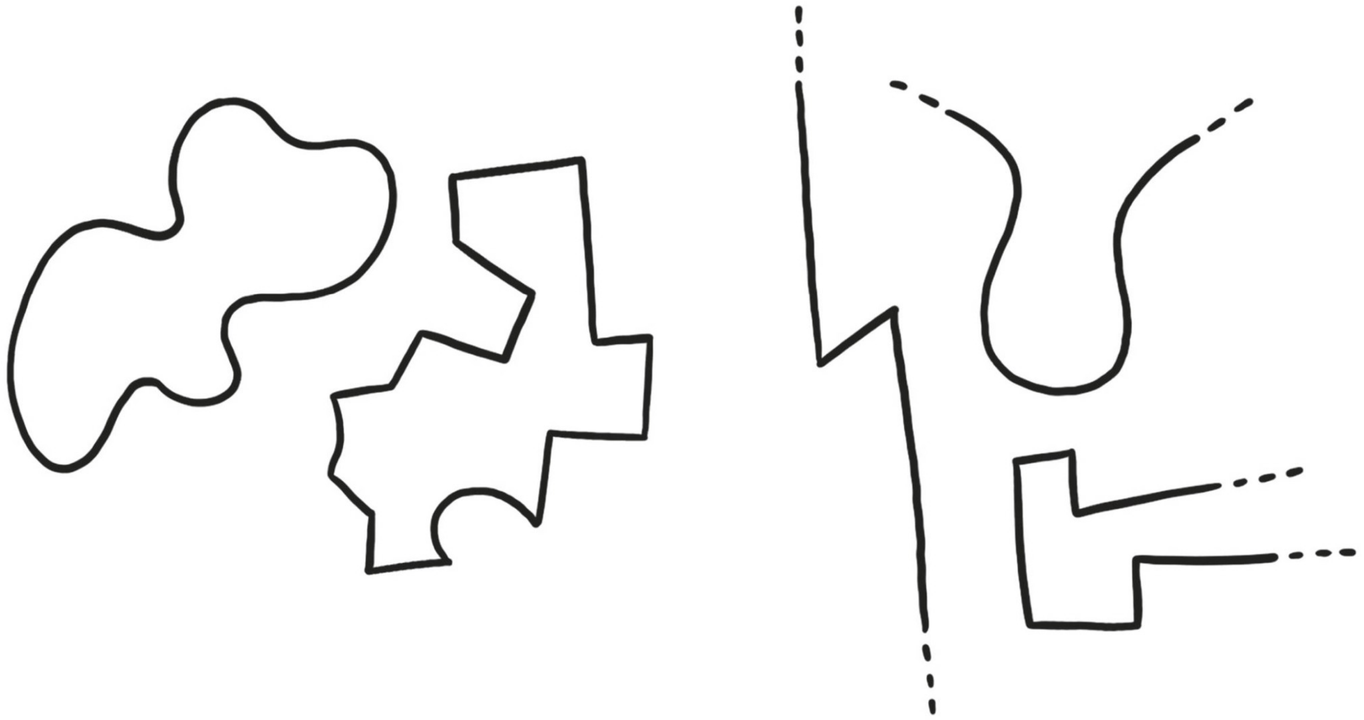
HINT: NO ENDPOINTS



all possible
string - manifolds

R-1 & S-1

- **Circle - S-1**
- **Infinite line – R-1**

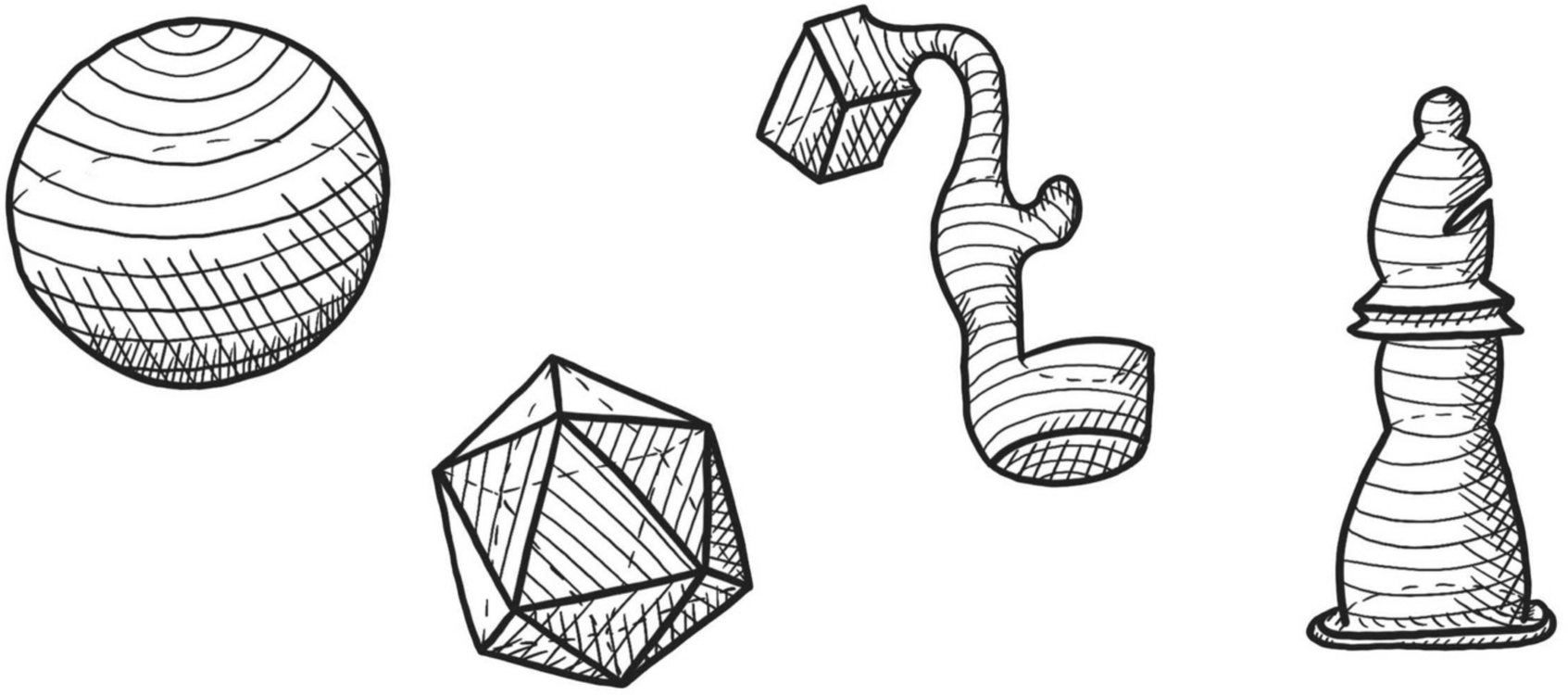


TWO-DIMENSIONAL MANIFOLD?

MADE BY SHEET-LIKE MATERIAL

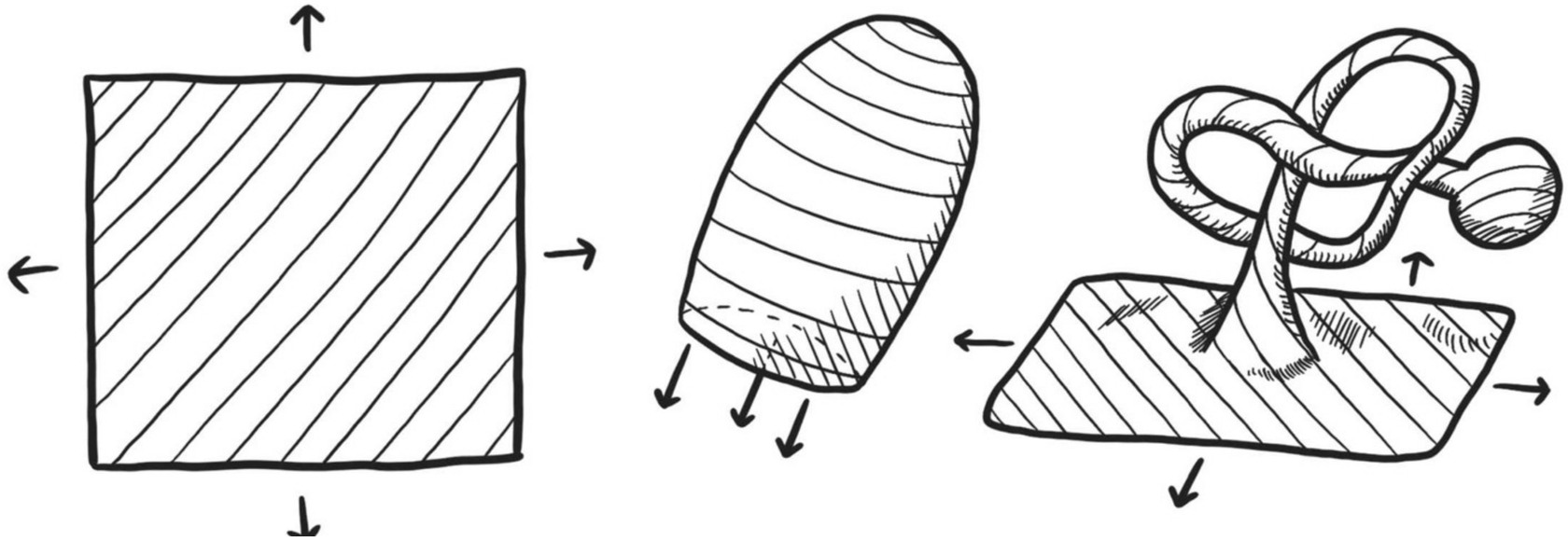
HINT: WE LIVE ON A MANIFOLD, SIMILAR LOGIC AS 1-D CASE

S-2



R-2

- **Any infinite surface that divides space into two infinite regions.**



EARTH IS FLAT?

- **Manifolds do not have special point**
- **Would feel like living on a plane if you lived on any sheet-manifold**

MORE 2-D MANIFOLDS!

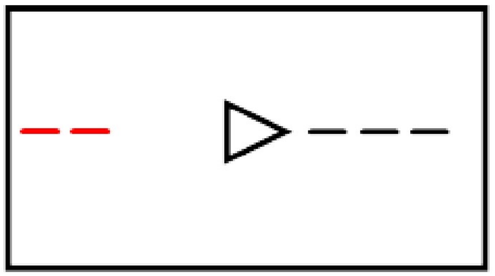
IDENTITY SPACE

GAME ASTEROIDS

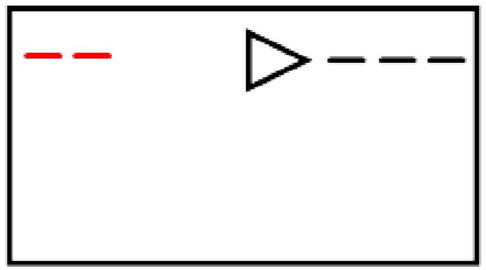
- https://www.youtube.com/watch?v=5rjjtJ2GMN8

- **Questions to consider while watching:**
- **What shape appears to be the space for the game?**
- **What happened when an object came into contact with one of the sides of the shape?**

a)

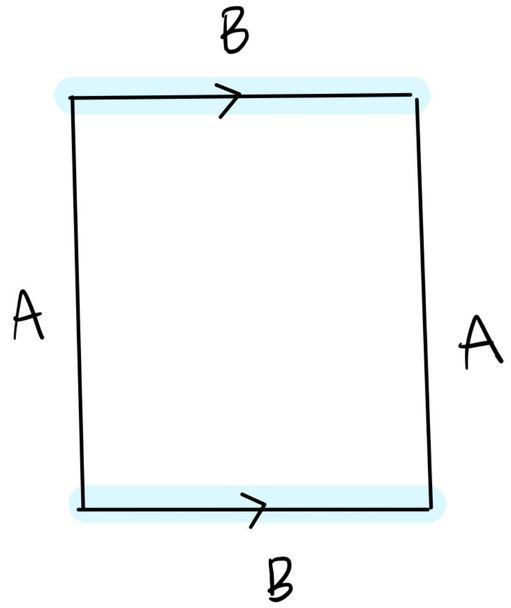


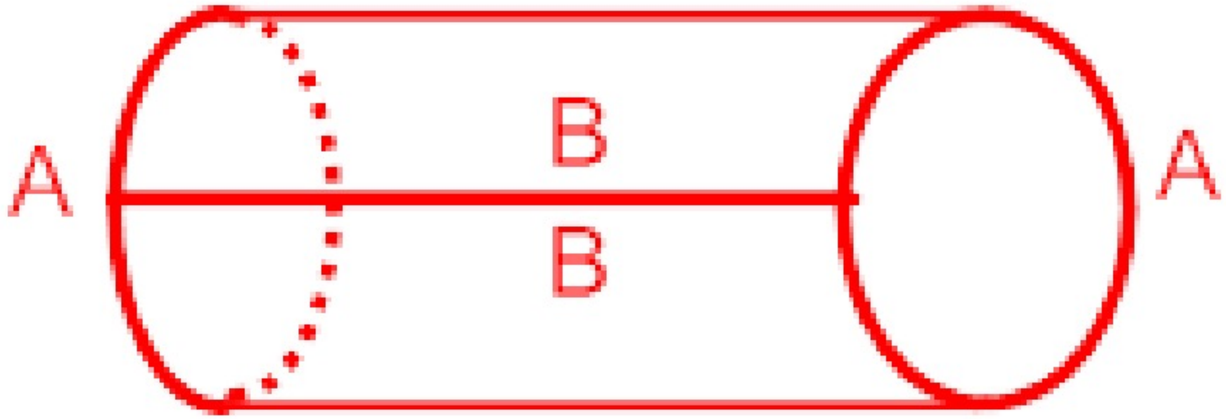
b)



GLUE SIDES TOGETHER

- **If you were given a piece of paper, how could we recreate what happens to the objects in the game Asteroids?**
- **The arrows represent the gluing of the shape. You must think of the gluing "matching" up the arrows.**





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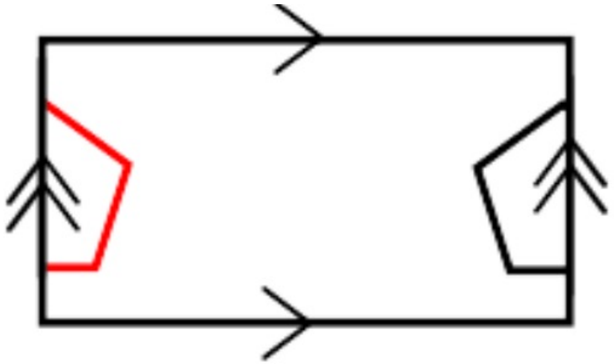
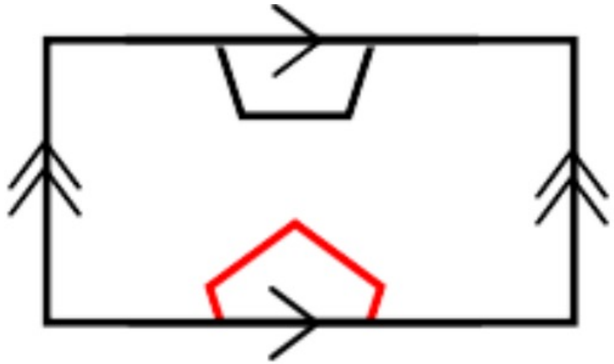
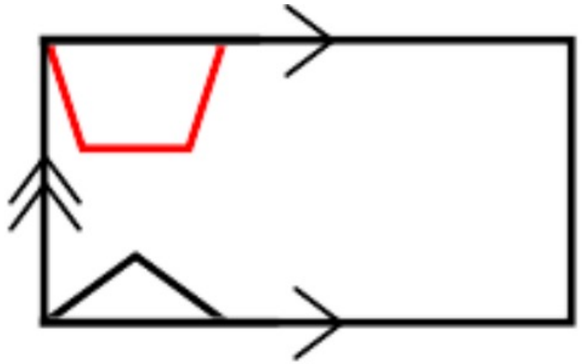
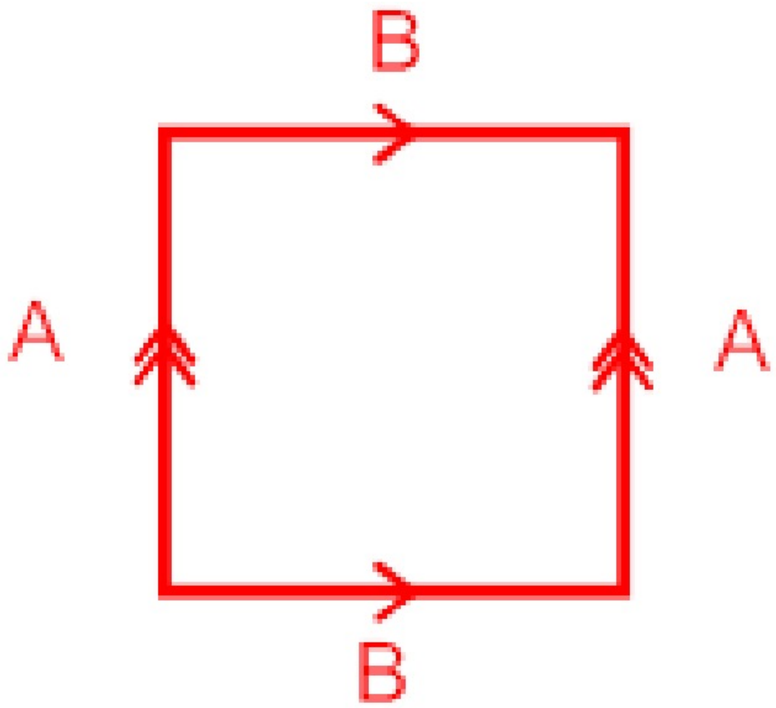
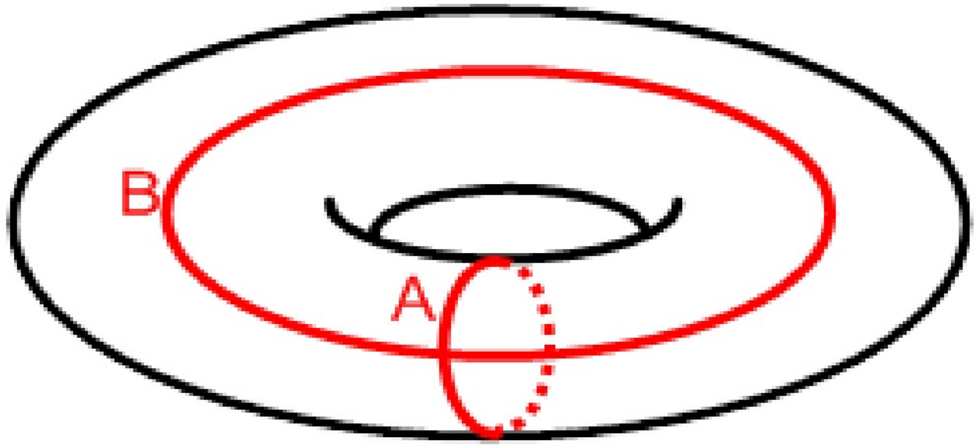






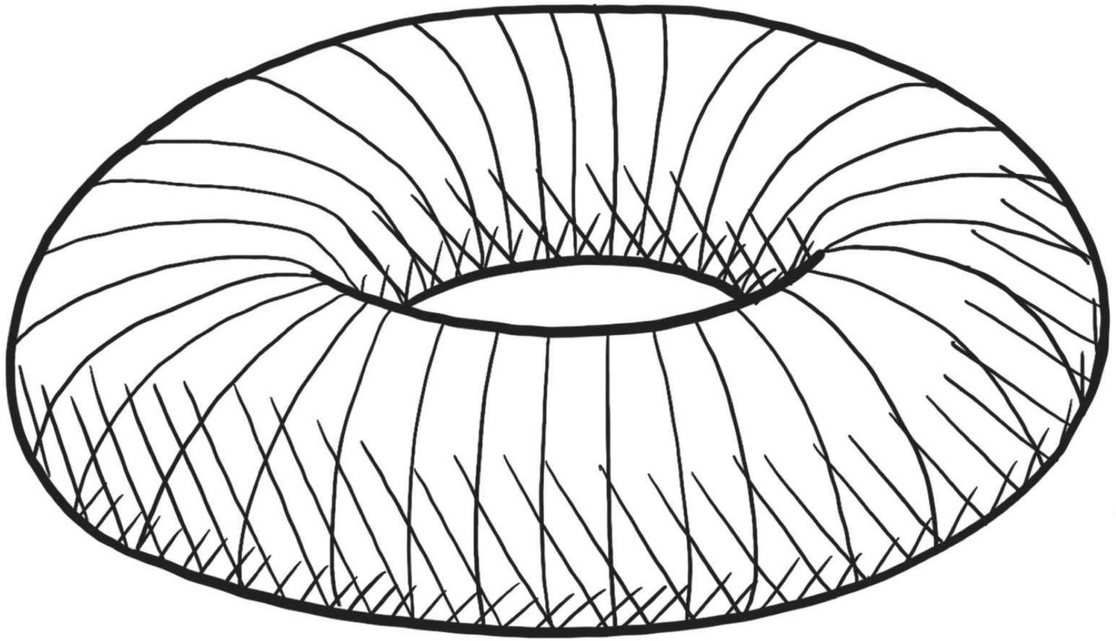
WHAT IS THIS MANIFOLD

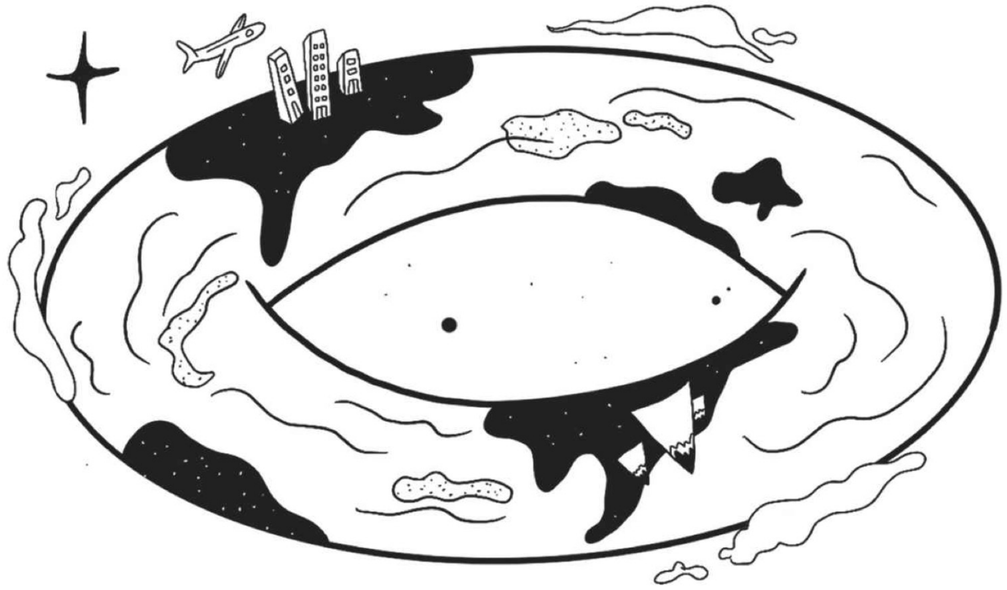




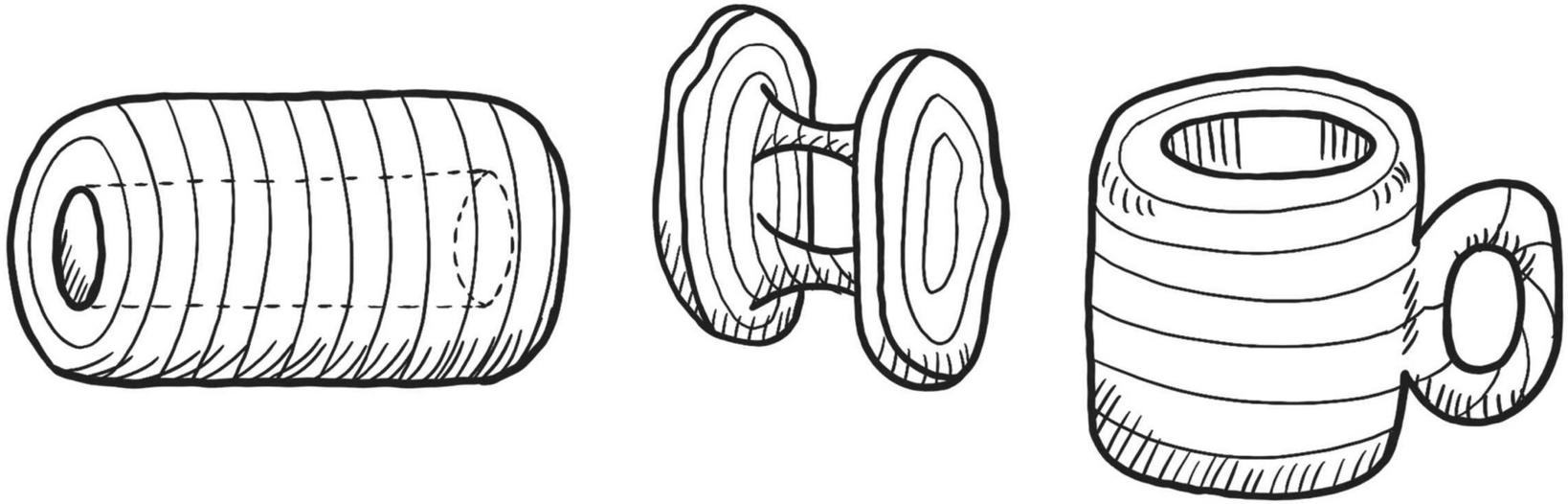
DONUT

- **If you lived on the surface of a donut-shaped planet, you'd never notice from looking around that there was a hole. It would look, locally, just like if you lived on a sphere or a flat plane.**

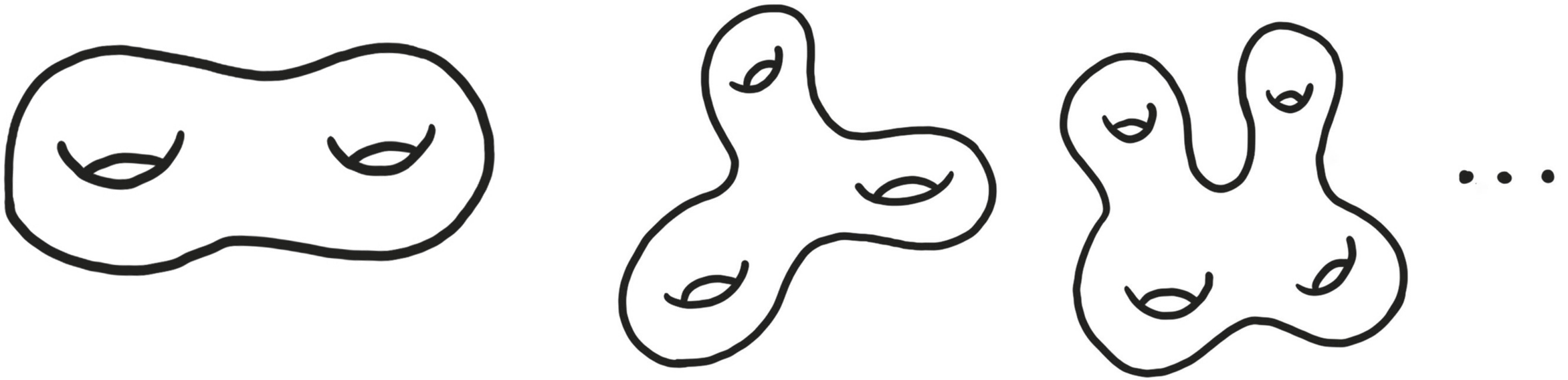




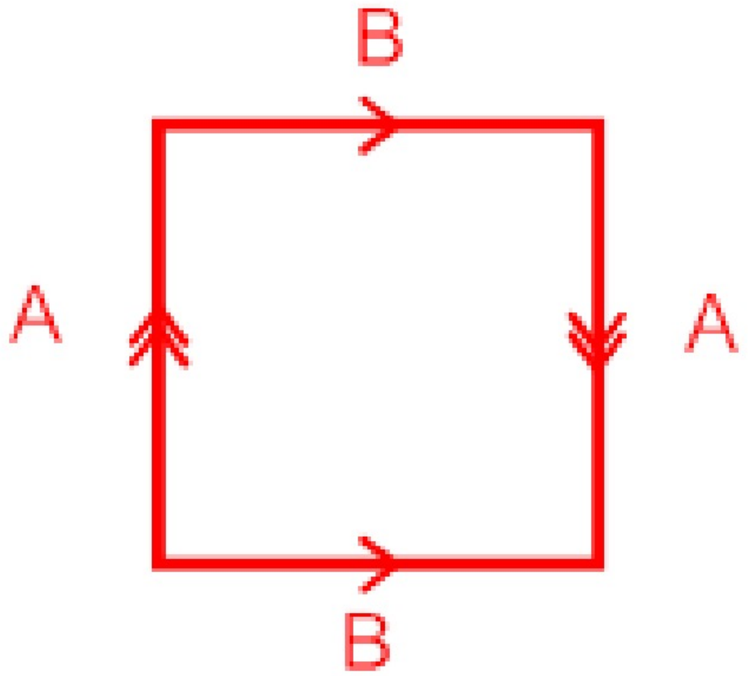
T-2



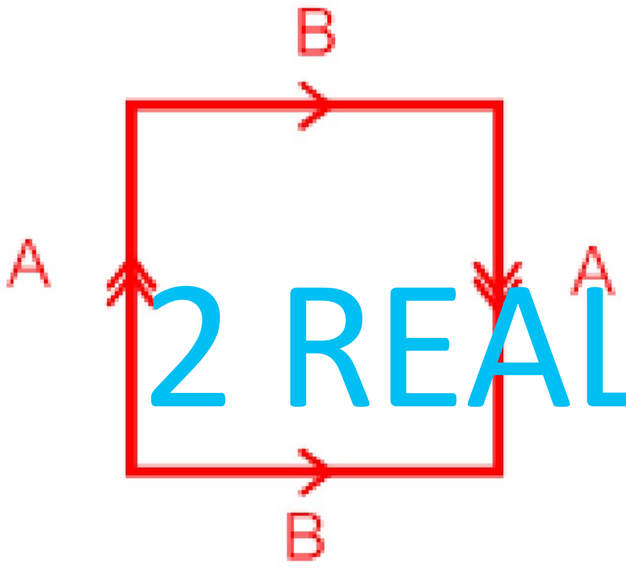
MORE TORI

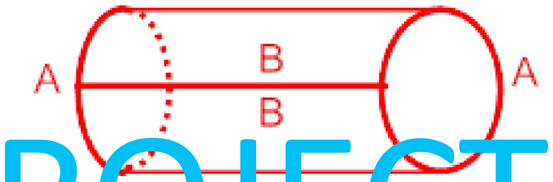


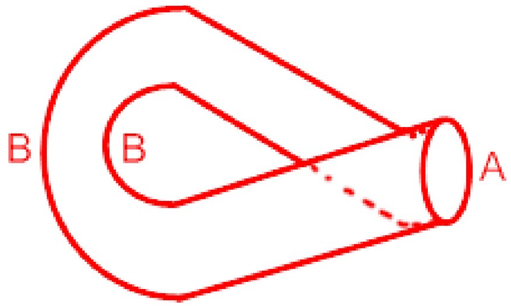
WHAT IS THIS IN 2D?

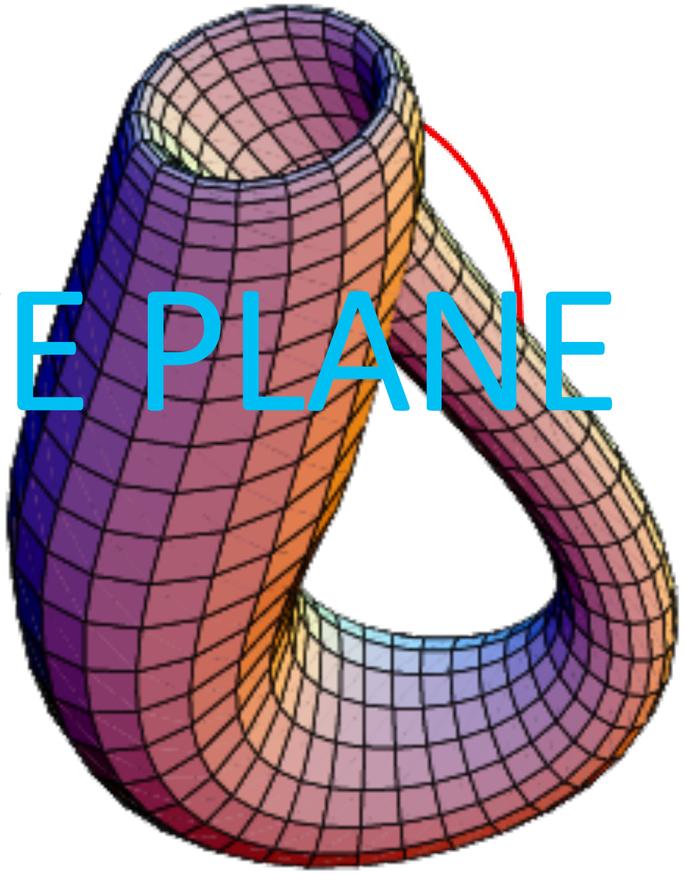


KLEIN BOTTLE



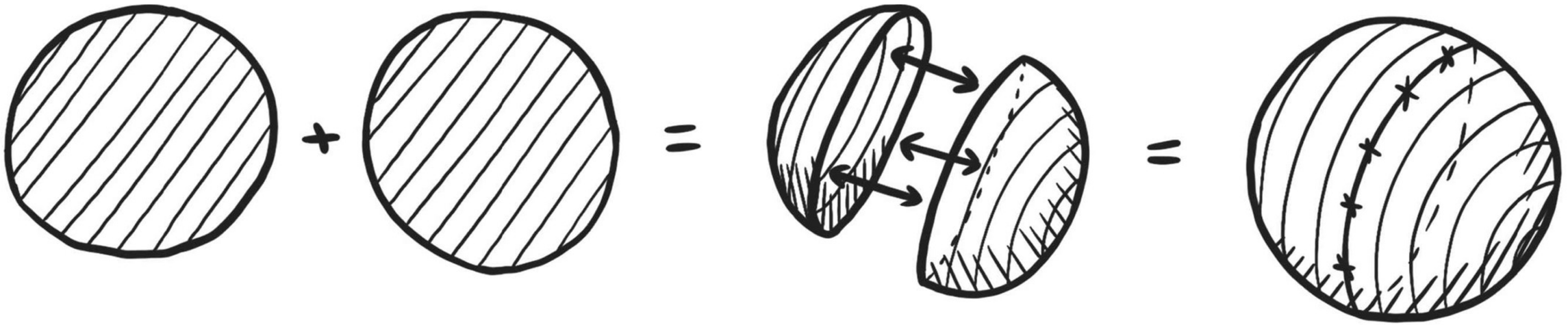




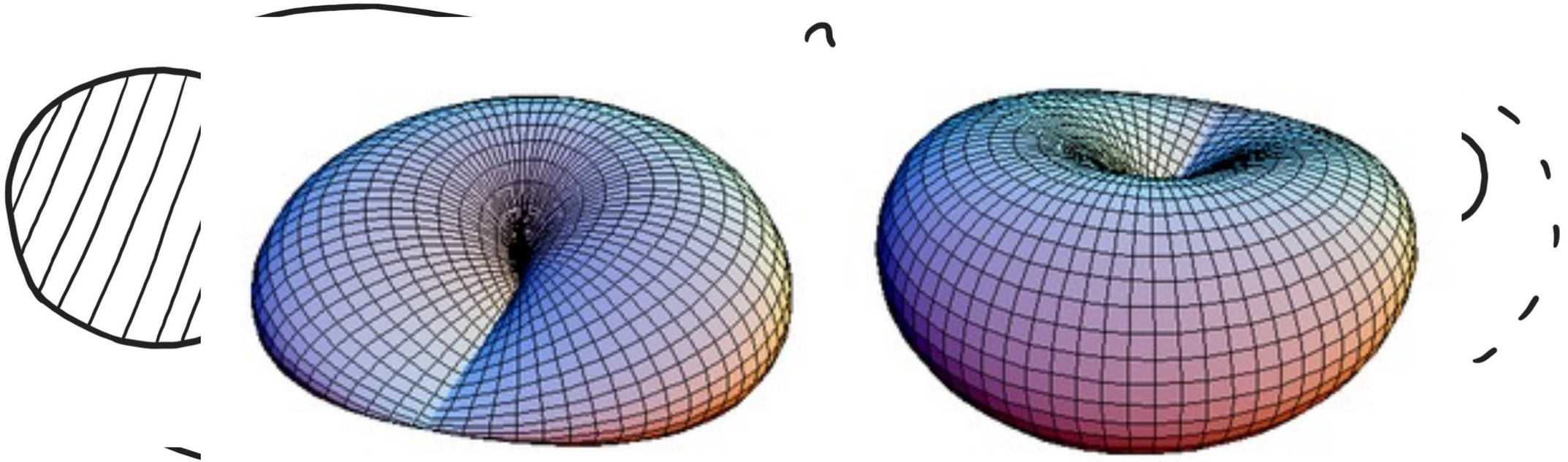


2 REAL PROJECTIVE PLANE

SPHERE



RP2 REAL PROJECTIVE PLANE



EXIST IN 4-D?

- 8

R-2

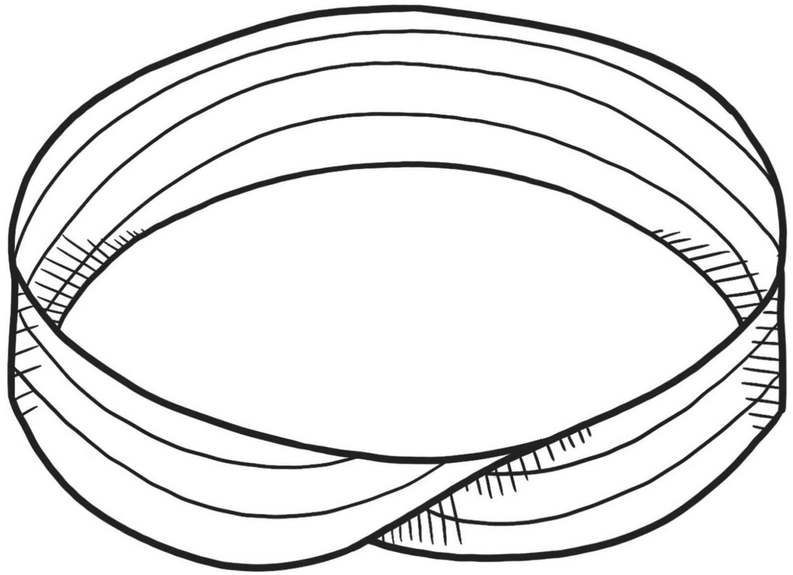
- only one side: A sphere and a torus have an inside and an outside, but a real projective plane just has one side that twists to the inside and out.
- If you write the letter R on a sphere or torus, and slide it around through the space, it'll always come back looking like an R. But if you slide an R around on a real projective plane, it could come back looking like an inverse R.

ORIENTABILITY

DEFINITION

- **A surface is orientable if a figure making all possible global trips on the surface does not change its orientation at any time.**

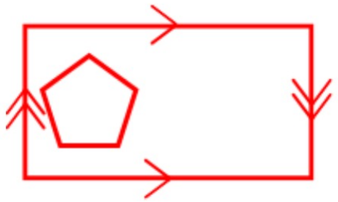
- **Möbius Strip https://www.youtube.com/watch?v=JBYXT9AAOvc 3:22**



KLEIN BOTTLE

- **Would this happen if the pentagon moved up and down instead of left and right?**
- **orientation is preserved.**
- **What would happen if it went through the right side?**

1.



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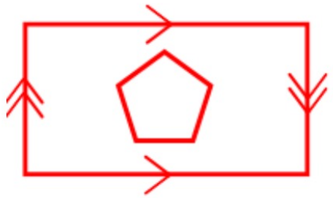

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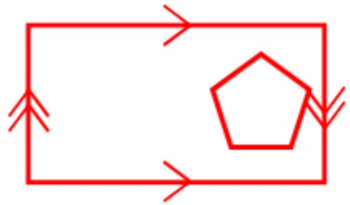

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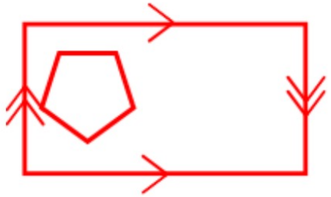

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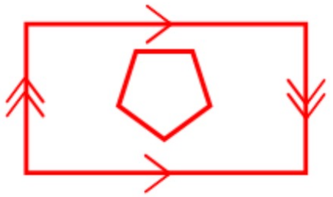
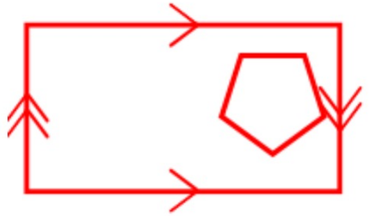
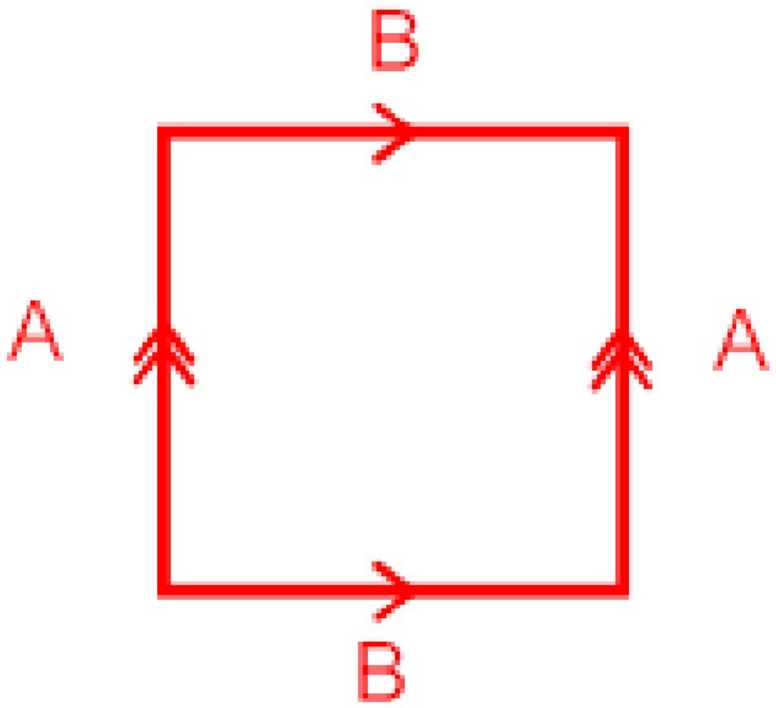


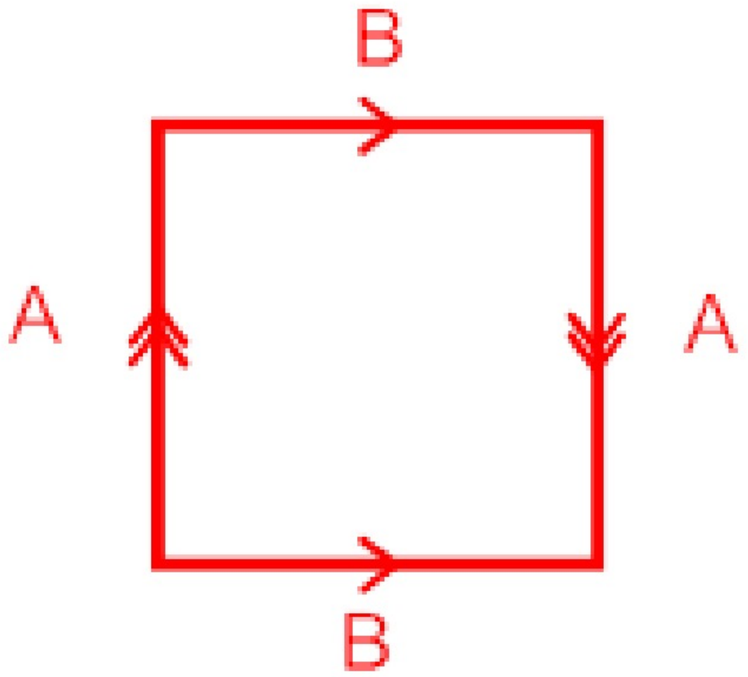
6.



WHAT IS THE DIFFERENCE BETWEEN THE TORUS AND THE KLEIN BOTTLE?

- **One of the pairs of glued sides has the arrows going in the opposite direction**





HIGHER DIMENSIONS

- 3- dimension: dough-type manifolds
- dimensions 5 and up: “surgery theory”
- dimension 4